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GLOBAL STABILITY ANALYSIS OF BIRHYTHMICITY IN A VAN DER POL TYPE SELF-SUSTAINED OSCILLATOR

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The properties of a multi-limit cycles variation of the van der Pol self sustained oscillator with random excitations are reviewed [7, 8, 9]. Birhythmicity refers to the coexistence of two attractors characterized by two different amplitudes and two frequencies: depending on the initial conditions, the system can produce self–oscillations at two distinct periods [4]. Such hysteretic behavior has been sometimes observed in biological systems [1], glycolytic oscillations [4] or circadian rhythms [6] . As a prototype of birhythmic oscillations, we consider a model that has been introduced to analyze enzymatic substrate reactions in brain [2, 3, 10]. The global stability properties of birhythmicity can be investigated through the effect random excitations, such as a Gaussian white noise, on the attractors global stability. Noise makes the two limit cycles metastable, and induces switches between the two attractors. The mean escape time from each limit-cycle can give an estimate of the effective activation energy barrier through the slope of the (log of the) escape time as a function of the inverse of the noise intensity. Several properties can be retrieved, most importantly that the trapping barriers of the two frequencies can be very different, thus leaving the system on the same attractor for an overwhelming time. We conclude that although birhythmicity *per se* refers just to the occurrence of two frequencies, actual observation is subject to much more restrictive conditions. Moreover, the system is analytically particularly tractable by means of stochastic averaging; thus the effect

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of external periodic drive, noise correlation and the coherence of the oscillations, or time delayed feedback, can be analyzed [1, 7, 8, 9, 10].

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