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ALLEE EFFECT BIFURCATION IN THE γ-RICKER POPULATION MODEL USING THE LAMBERT W FUNCTION

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The classical discrete Ricker population model was proposed for modeling fish populations, see [1]. Since then this overcompensatory model has been used in several studies with applications to different types of populations, see, for example, references in [2]. In the definition of the classical discrete Ricker population model, given by the difference equation $x_{n+1} = r\, x_n\, e^{-\delta x_n}$, is assumed that the survival function for generation n is density-dependent, while the birth or growth rate is density-independent. However, in several applications of this model to biology and ecology there are circumstances which lead to non constant density-dependent birth or growth functions. This phenomenon can be caused by several factors : difficulty to find mates, environmental modification, predator satiation, cooperative defense, among others. This model is classified in several studies as relatively inflexible, since it has only two parameters.

In this work it is considered the discrete-time population model whose dynamics of the population x_n, after n generations, is defined by the difference equation,

$$x_{n+1} = b(x_n)\, x_n\, s(x_n), \text{ with } n \in \mathbb{N} \tag{0.1}$$

where $b(x_n) = x_n^{\gamma-1}$ is the per-capita birth or growth function (a cooperation or interference factor), with $\gamma > 0$ the cooperation parameter or Allee effect parameter, $s(x_n) = e^{\mu-\delta x_n}$ is the survival function for generation n or the intraspecific competition, with $\mu > 0$ the density-independent death rate and $\delta > 0$ the carrying capacity parameter. In particular, we consider the γ-Ricker population model defined by Eq.(0.1) written in the form,

$$x_{n+1} = r\, x_n^{\gamma}\, e^{-\delta x_n} := f(x_n) \tag{0.2}$$

where $r = e^{\mu}$, γ and δ are positive real parameters and $f : [0, +\infty[\to [0, +\infty[$. This stock-recruitment model is usually called γ-Ricker model or γ-Ricker map. The particular case $\gamma = 1$ is known as the classical overcompensatory Ricker model, which was introduced by Ricker in the context of stock and recruitment in fisheries, see [1]. The limit case $\delta \to 0^+$ corresponds to the Cushing model. Throughout this work, the parameters space is denoted by,

$$\Sigma_0 = \left\{(r, \gamma, \delta) \in \mathbb{R}^3 :\ r, \gamma, \delta > 0 \text{ and } \gamma \neq 1\right\}. \tag{0.3}$$

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The main purpose of this talk is to present the dynamical study and the bifurcation structures of the γ-Ricker population model. Resorting to the Lambert W function, the analytical solutions of the positive fixed point equation for the γ-Ricker population model are explicitly presented and conditions for the existence and stability of these fixed points are established. The use of the Lambert W function, generally defined as the real analytic inverse of the function $W(x) = xe^x$, allows us to obtain a deeper insight and a new point of view of the behavior of the γ-Ricker population model. This procedure proves to be extremely useful, since the fixed points expression of the γ-Ricker population model is an implicit condition.

Another main focus of this work is the definition and characterization of the Allee effect bifurcation for the γ-Ricker population model, which is not a pitchfork bifurcation. Consequently, we prove that the phenomenon of Allee effect for the γ-Ricker population model is associated to the asymptotic behavior of the Lambert W function in a neighborhood of zero. The theoretical results describe the global and local bifurcations of the γ-Ricker population model, using the Lambert W function in the presence and absence of the Allee effect. The Allee effect, *snap-back repeller* and big bang bifurcations are investigated in the parameters space considered. Numerical studies are included.

Références

[1] W.E. Ricker. (1954). *Stock and recruitment*, J. Fish. Res. Board Canada, **11 (5)**, 559–623.

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