11th Conference on Dynamical Systems Applied
to Biology and Natural Sciences DSABNS 2020
Trento, Italy, February 4–7, 2020

BIFURCATIONS IN REACTION CROSS-DIFFUSION SYSTEMS FOR COMPETING SPECIES

M. Breden[1], C. Kuehn[2] and C. Soresina[*2]

[1]CMAP, École Polytechnique, France

[2]Zentrum Mathematik, Technische Universität München, Germany

maxime.breden@polytechnique.edu, ckuehn@ma.tum.de
soresina@ma.tum.de (*corresponding author),

The Shigesada–Kawasaki–Teramoto model (SKT) was proposed to account for stable inhomogeneous steady states exhibiting spatial segregation [4], which describes a situation of coexistence of two competing species. Despite the fact that the reaction part does not present the activator–inhibitor structure, the cross-diffusion terms are the key ingredient for the appearance of spatial patterns. We provide a deeper understanding on the conditions required on both the cross-diffusion and the reaction coefficients for non-homogeneous steady states to exist, by combining a detailed linearized analysis with advanced numerical bifurcation methods via the continuation software `pde2path` [5]. We report some numerical experiments suggesting that, when cross-diffusion is taken into account, there exist positive and stable non-homogeneous steady states outside of the range of parameters for which the coexistence homogeneous steady state is positive [1]. In 1D and 2D, we pay particular attention to the fast-reaction limit [2] by computing sequences of bifurcation diagrams as the time-scale separation parameter tends to zero. We show that the bifurcation diagram undergoes major deformations once the fast-reaction systems limits onto the cross-diffusion singular limit. Furthermore, in 2D we find evidence for time-periodic solutions by detecting Hopf bifurcations, we characterize several regions of multi-stability, and improve our understanding of the shape of patterns [3].

References

[1] Breden, M., Kuehn, C., & Soresina, C. (2019). *On the influence of cross-diffusion in pattern formation*. arXiv preprint 1910.03436.

[2] Iida, M., Mimura, M., & Ninomiya, H. (2006). *Diffusion, cross-diffusion and competitive interaction*. Journal of Mathematical Biology, 53(4), 617–641.

[3] Kuehn, C., & Soresina, C. (2019). *Numerical continuation for a fast-reaction system and its cross-diffusion limit*. arXiv preprint 1911.01906.

[4] Shigesada, N., Kawasaki, K., & Teramoto, E. (1979). *Spatial segregation of interacting species*. Journal of Theoretical Biology, 79(1), 83–99.

[5] Uecker, H., Wetzel, D., & Rademacher, J. D. (2014). `pde2path` — *A Matlab package for continuation and bifurcation in 2D elliptic systems*. Numerical Mathematics: Theory, Methods and Applications, 7(1), 58–106.

 ISBN: 978-989-98750-7-4