

OPTIMAL YIELD AND UTILITY IN STOCHASTIC BIOMASS MODELS

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Optimal management of exploited ecosystems such as fisheries is a subject in theoretical ecology of great interest to society. Here, we consider the problem of optimal harvest strategies in a stochastic continuous-time biomass model of a fish stock; specifically, the stochastic logistic growth model and a variant thereof, the Pella-Tomlinson model. Similarly to previous related works (e.g. [1, 2] and references therein) we frame the problem as one of optimal control for stochastic differential equations and use the stationary Hamilton-Jacobi-Bellman equation to identify the optimal Markov (i.e., state feedback) strategy. We first consider the problem of maximizing the yield and extend recent results [2] regarding the resulting “bang-bang” policy where the fishery is either closed or unrestricted, depending on the current biomass. Next, we consider concave utility functions. In the case of stochastic logistic growth and a square root relationship between catch and utility, the optimal solution is found analytically: The optimal fishing effort is proportional to the biomass and is independent of the intensity of the process noise, which however reduces the expected profit in steady state. We then consider the Pella-Tomlinson model, where solutions must be found numerically, and show that the asymmetry in the growth curve is also found in the resulting optimal harvest strategy. We finally extend the system to two coupled stochastic differential equations, describing predator-prey dynamics, where each species is harvested. For this system solutions must be found numerically. The optimal harvest policies depend on the relative value of predators vs. prey but tend to remove predator-prey cycles in the unfished system.

References

- [1] Nuno M Brites and Carlos A Braumann. (2017). *Fisheries management in random environments: Comparison of harvesting policies for the logistic model*. Fisheries Research, 195:238–246.
- [2] Alexandru Hening, Dang H. Nguyen, Sergiu C. Ungureanu, and Tak Kwong Wong. (2019). *Asymptotic harvesting of populations in random environments*. Journal of Mathematical Biology, 78(1):293–329.