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MATHEMATICAL MODELLING OF SELECTION BY A SEX-LIMITED FEMALE TRAIT: TO THE QUESTION OF LITTER SIZE POLYMORPHISM IN NATURAL POPULATIONS OF ARCTIC FOXES

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Sex-limited genes are present in both sexes of sexually reproducing species but expressed in only either sex and remain turned off in the other. These traits include animal productivity, such as milk production level and milk fattiness in cattle, egg production and egg size in chicken, and litter size in arctic foxes [1]. Classic equations of mathematical population genetics are inappropriate for such genes because both parents define progeny genotypes, whereas fitness depends only on the mothers genotype. As a result, such a population is not in a HardyWeinberg equilibrium (figure). Therefore, one needs to develop an evolution model in equations for genotype frequencies instead of allele frequencies as in the classic theory of natural selection.

Our study proposed models of natural selection of the litter size trait, which is controlled by a sex-limited gene that is expressed only in females, to explain existing genetic variations in litter sizes in coastal, inland, and farmed arctic fox populations. We compared the possibilities of maintaining polymorphism in the framework of the classic theory of modeling with the case of selection by a sex-limited gene.

For now, whether a system of balanced genetic polymorphisms of litter size are maintained in natural populations remains an open question [1], because the populations of inland foxes might have fixed the dominant allele. Thus, one may suppose that the farmed foxes inherited the alleles for small and large litter size from coastal and inland populations, respectively. Alternatively, and more interestingly, they might inherit both alleles from the inland foxes providing that a system of balanced genetic polymorphism of litter size was maintained in the natural populations. However, the conditions for polymorphism maintenance in the classic model are restrictive for the range of values that characterizes the advantage of survival in small litters in years with low food supply.

Our study considering the inheritance type of this trait (i.e. sex-limited gene) allows to wide a range of the values resulted in the maintenance of polymorphism. Furthermore, it is known that the ability of fluctuations in selection to maintain genetic diversity is greatly increased when generations overlap. Generalization

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of the proposed model of natural selection on a population with two ages revealed that polymorphism can be maintained in an even wider range of parameter values. Thus, this study shows that selection by a sex-limited gene and overlapping generations increases the chances of maintaining polymorphism in inland populations of arctic foxes.

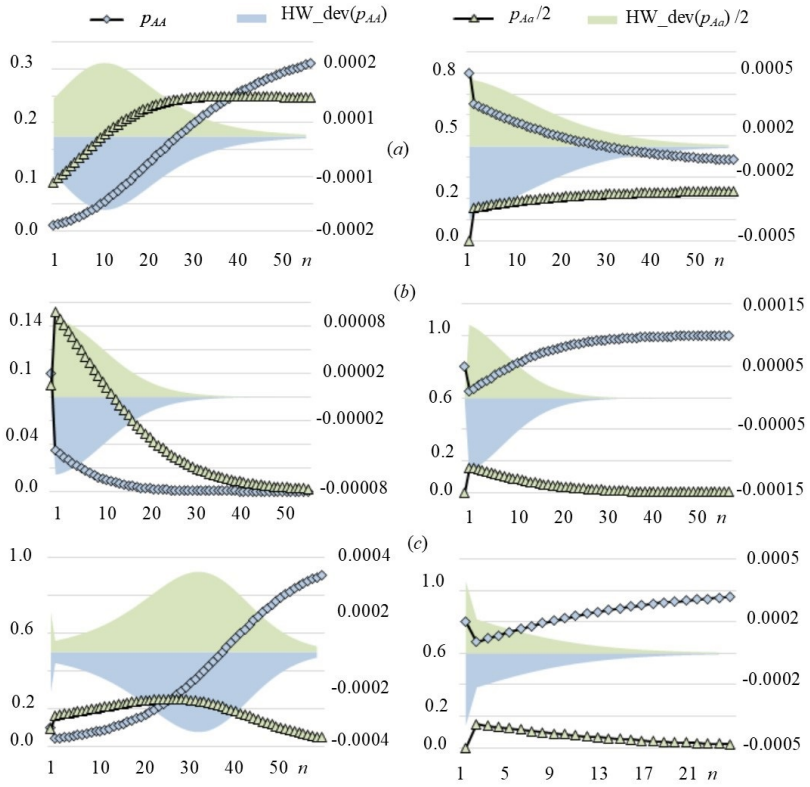

Figure 1: Dynamics of the frequencies of genotypes (p_{AA} and p_{Aa}, the left axis) and their deviations from HWE ($HW_{dev}(p_{AA})$ and $HW_{dev}(p_{Aa})$, the right axis) over generations under various types of selection.

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