

# Mathematical modeling physiological tick life cycle and interaction with epidemiology

Slimane Ben Miled

Joint work with Mamadou Sadio Ndong, Papa Ibrahima Ndiaye,  
Mohamed Gharbi, Mourad Rekik and Mohamed Aziz Darghouth  
[slimane.benmiled@fst.utm.tn](mailto:slimane.benmiled@fst.utm.tn)

Institut Pasteur de Tunis, Tunis el Manar University



DYNAMICAL SYSTEMS APPLIED TO BIOLOGY AND  
NATURAL SCIENCES (DSABNS)  
Trento, Feb. 4-7, 2020



- *Theileria annulata* is an apicomplexan protozoan parasite of cattle transmitted by ticks. The parasite causes an acute, often fatal disease (tropical theileriosis).
- Compared to other tick-transmitted diseases, tropical theileriosis poses a particularly serious constraint to livestock production due to the lack of resistance<sup>1</sup>.

---

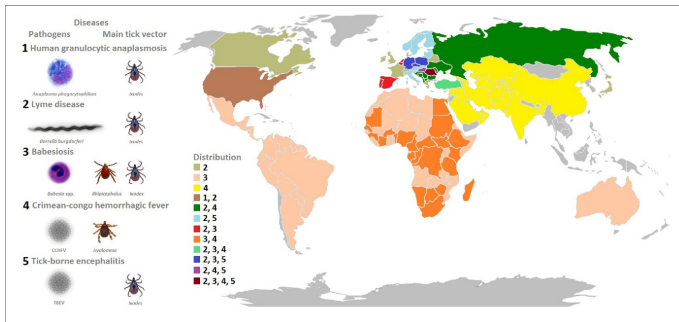
<sup>1</sup>Gharbi M, Sassi L, Dorchies P, Darghouth MA. Infection of calves with *Theileria annulata* in Tunisia: Economic analysis and evaluation of the potential benefit of vaccination. *Vet. Parasitol.* 2006, 137(3-4):231-241.

# Word distribution of Theileria

*Distribution of major Theileria species of cattle*



# Tick-Pathogen Interactions and Vector Competence:



From Jos de la Fuente, et al. Published in Front. Cell. Infect. Microbiol. 2017 *Tick-Pathogen Interactions and Vector Competence: Identification of Molecular Drivers for Tick-Borne Diseases*.

# Goal

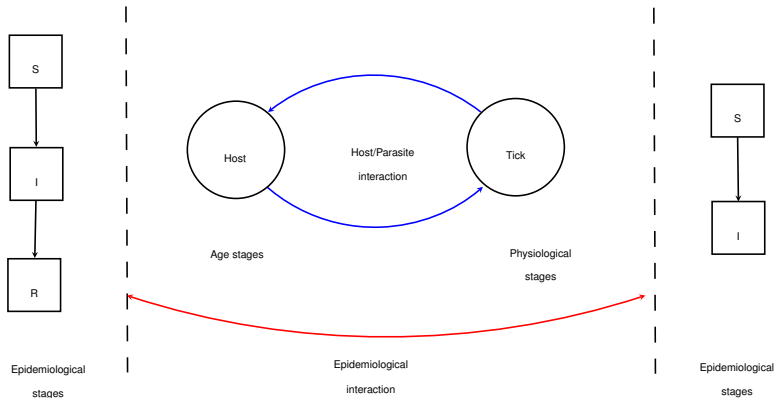
## Ultimate goal

- 1 To study climate change on transmission dynamics of tick-borne disease:
  - Cattle population
  - Vector (tick) population
- 2 Investigate the impact of a range of hypothetical interventions:
  - On transmission.
  - On cost effectiveness.

## Goal of this work

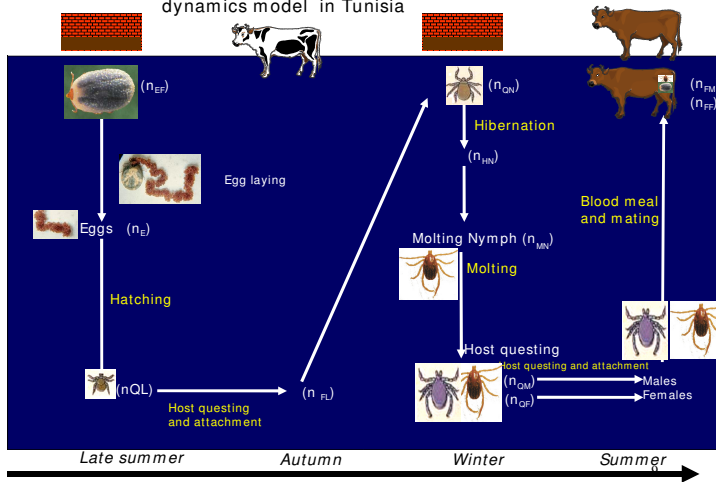
Study impact of climate change on tick life cycle duration, taking account physiological stages.

# Level of complexity

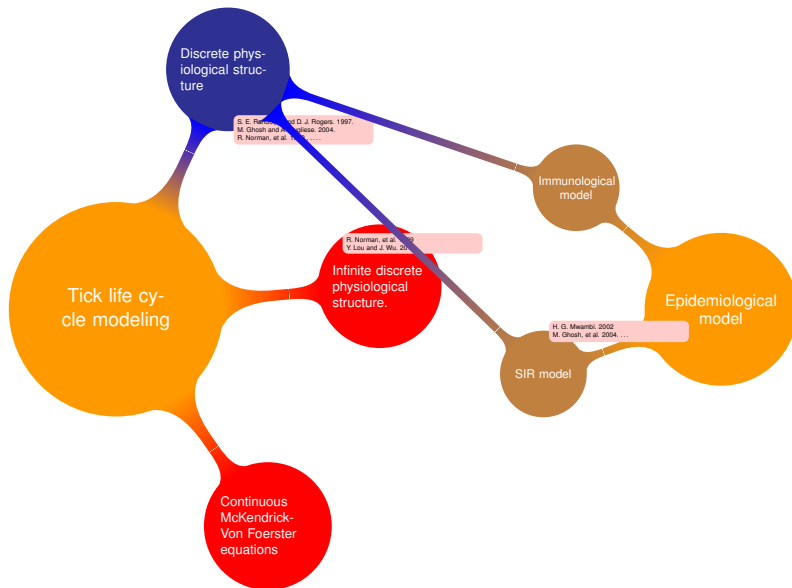


# Life Cycle

Compartmental model for *Hyalomma detritum* activity  
dynamics model in Tunisia

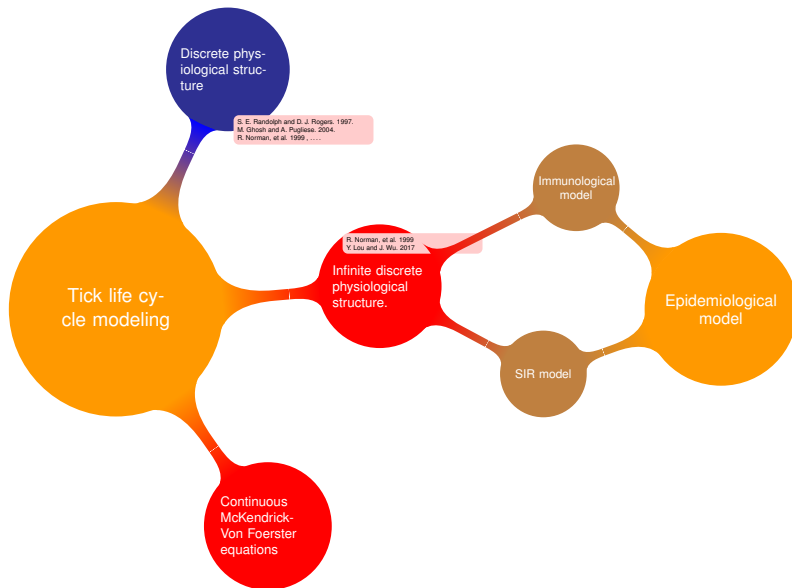


# Step to achieve to goal

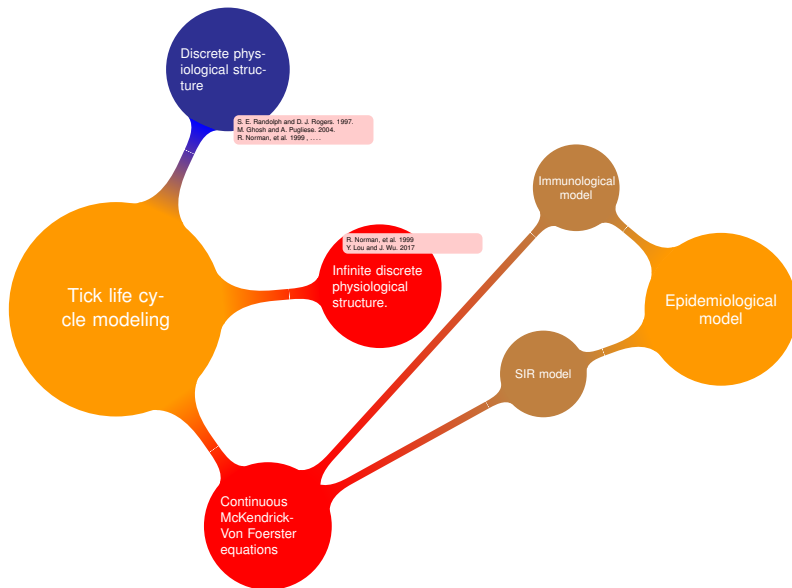


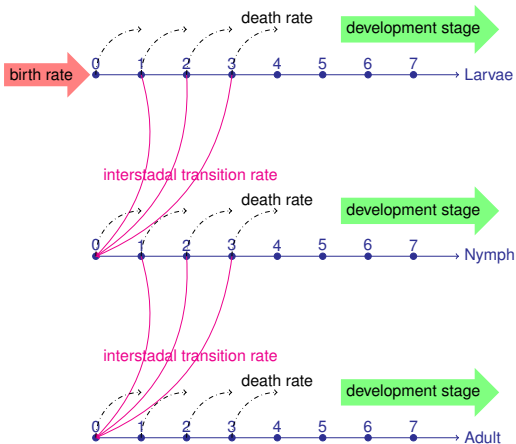


# Step to achieve to goal



# Step to achieve to goal



**Subprocesses**

death

birth

dev. stage

inter. transition

**Dependence**Stage, temperature  
and precipitation

Temperature

Temperature

Stage

**Figure:** A schematic diagram for the model, identifying the input parameters and the model output (predicted number of ticks of each stage).

# Model equations:

Larvae density

**Larvae:**

$$\begin{cases} \frac{d l_0(t)}{dt} = \sum_{i=1}^{+\infty} \beta^i a_i(t) - \mu_l^0 l_0(t) - g_l^0 l_0(t), \\ \frac{d l_i(t)}{dt} = -(T_l^i + \mu_l^i + g_l^i) l_i(t) + g_l^{i-1} l_{i-1}(t), \quad i \geq 1. \end{cases}$$

Development stage,  $i \in \mathbb{Z}_+$

Interstate transition rate

Nymph density

**Nymph:**

$$\begin{cases} \frac{d n_0(t)}{dt} = \sum_{i=1}^{+\infty} T_n^i l_i(t) - \mu_n^0 n_0(t) - g_n^0 n_0(t), \\ \frac{d n_i(t)}{dt} = -(T_n^i + \mu_n^i + g_n^i) n_i(t) + g_n^{i-1} n_{i-1}(t), \quad i \geq 1. \end{cases}$$

Tick mortality rate

Intrastate transition rate

Adult density

**Adult:**

$$\begin{cases} \frac{d a_0(t)}{dt} = \sum_{i=1}^{+\infty} T_a^i n_i(t) - \mu_a^0 a_0(t) - g_a^0 a_0(t), \\ \frac{d a_i(t)}{dt} = -(\mu_a^i + g_a^i) a_i(t) + g_a^{i-1} a_{i-1}(t), \quad i \geq 1. \end{cases}$$

## Well-posedness of the Model

Under assumptions of bounded function,  $A$  is generator of an infinitesimal positive  $C_0$ -semigroup,  $(T_A(t))_{t \geq 0}$ , on  $E$ .

## Basic reproduction number formula

$$R_0 = f_l(0)f_n(0) \sum_{i=1}^{+\infty} \frac{\beta^i}{\mu_a^i + g_a^i} \prod_{k=0}^{i-1} \frac{g_a^k}{\mu_a^k + g_a^k}$$

with

$$f_h(\lambda) = \sum_{i=1}^{+\infty} T_h^i \frac{\prod_{k=0}^{i-1} g_h^k}{\prod_{k=0} (\lambda + \tau_h^k + \mu_l^k + g_h^k)}, \text{ for } h = l, n.$$

We simulate ticks population of **larvae**, **nymph** and **adult** densities for a finite physiological structure  $(s_i)_{i \leq n}$  ( $n = 10$ ), and reproductive number of the population,  $R_0$ , for three different climate conditions:

- Tropical climate (Senegal),
- Mediterranean climate (Tunisia)
- Subartic climate (Canada)

and two different periods: 1901 – 1925 and 1991 – 2015 (bank data: Temperature and precipitation).

# Climate change independents parameters

- Interstate transition rate  $T_j^i$ : We suppose that **there is no blood meal limiting**, therefore,

For all  $i \leq n$  and  $j \in \{l, n\}$ :

$$T_j^i = T_{\max} \left( \frac{S_i - S_{\min}}{S_{\max}} \right)^\eta$$

with,  $\eta \in \mathbb{R}_+$  **a physiological parameter** and  $T_{\max}$  **the maximum transitions rate**.

- Larvae and nymph mortality,  $\mu_l^i$  and  $\mu_n^i$ : We will **overlook the fact that larvae and nymph mortality depends on temperature and precipitation** ([Randolph1997], in the case of *Rhipicephalus appendiculatus* in South Africa).

# Climate change dependents parameters

Climate parameters = Temperature,  $T$ , and precipitation,  $pr$ .

- Following [Randolph1997,Estrada-Pena2013], we suppose that both larvae hatching,  $\beta^i(T, pr)$ , and adult mortality rate depends on temperature and precipitation:

$$\beta^i(T, pr) = \begin{cases} \beta_{max} \exp(-0.594 + 0.016pr - 0.000215T) & \text{if } T \in [Tl, Th] \\ 0 & \text{if not} \end{cases}$$

with  $\beta_{max}$  the maximum hatching larvae.

- Adult mortality rate,  $\mu_a^i$ :

$$\mu_a^i = \check{\mu}_a + \exp(4.299 - 0.016T).$$



# Development stage dependents on Arrhenius function

Development stage,  $(g_j^i)_{i \leq n}$ , with  $j \in \{l, n, a\}$  depends on Arrhenius function.

For all time  $t$ , all  $i \leq n$  and  $j \in \{l, n, a\}$ :

$$g_j^i(t) = \hat{g}_j \dot{k}(T(t))$$

$\dot{k}$  the metabolic reaction rate  
(i.e. the Arrhenius function) at  
the absolute temperature (in  
Kelvin),  $T$

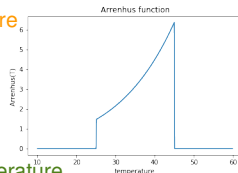
with  $\hat{g}_j$  is a normalising coefficients.

Metabolic reaction rate at  $T_1$

$$\dot{k}(T) = \begin{cases} \dot{k}_1 \exp\left(\frac{T_A}{T_1} - \frac{T_A}{T}\right) & \text{if } T \in [T_l, T_h] \\ 0 & \text{if not} \end{cases}$$

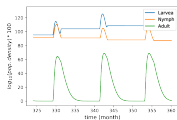
The Arrhenius temperature

A chosen ref. temperature

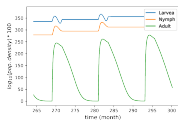


Where  $T \in [T_l, T_h]$  the temperature tolerance range bound.

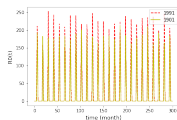
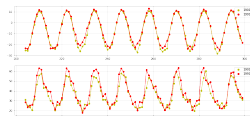
## Results



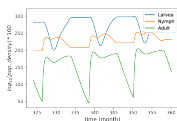
() CA-1901 – 1925



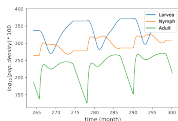
() CA-1991 – 2015

()  $R_0$ 

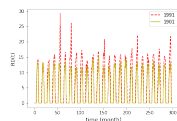
() Temp, pr



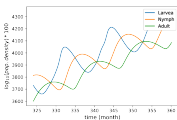
() TN-1901 – 1925



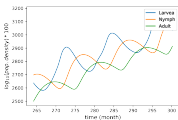
() TN-1991 – 2015

()  $R_0$ 

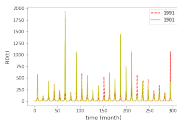
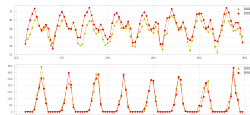
() Temp, pr



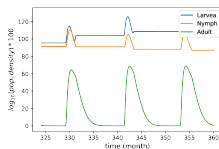
() SN-1901 – 1925



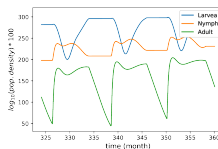
() SN-1991 – 2015

()  $R_0$ 

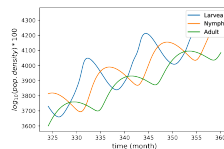
() Temp, pr



() CA-1901 – 1925

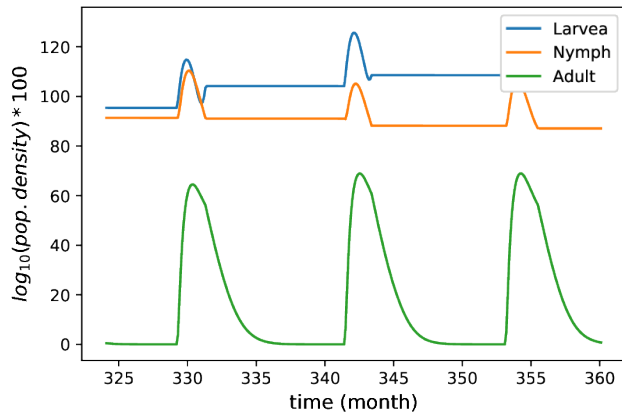


() TN-1901 – 1925

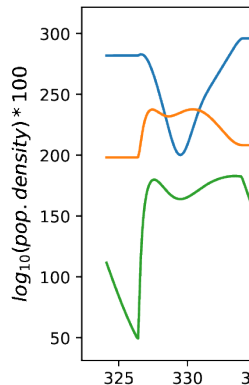


() SN-1901 – 1925

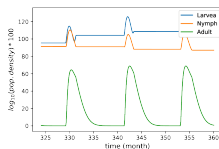
- 1 We notice that the densities of larvae are higher, than those of nymphs and adults.



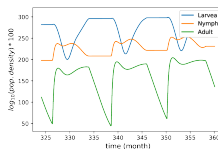
() CA-1901 — 1925



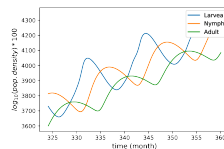
() TN-



() CA-1901 – 1925



() TN-1901 – 1925



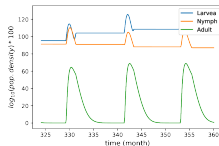
() SN-1901 – 1925

- 1 We notice that the densities of larvae are higher, than those of nymphs and adults.
- 2 This difference decreases as we move southward according to a positive temperature gradient from Canada to Senegal.

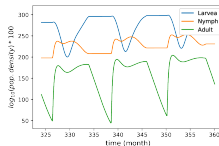
This observation is indicating the occurrence of a Northward-Southward temperature and precipitations seasonal gradient effect on tick mortalities.

## Results

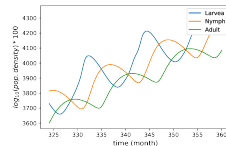
# There is a clear difference in the fluctuations of tick population densities between the three different countries.



$$() \text{ CA-} \frac{\text{adult}_{\text{summer}}}{\text{adult}_{\text{winter}}} = 160$$



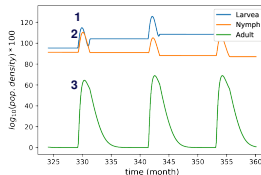
$$() \text{ TN-} \frac{\text{adult}_{\text{summer}}}{\text{adult}_{\text{winter}}} = 2$$



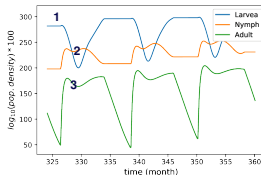
$$() \text{ SN-} \frac{\text{adult}_{\text{summer}}}{\text{adult}_{\text{winter}}} = 1.1$$

- In Canada: The adult population density varies between 0 in winter and becomes equal to 160 in summer.
- In Tunisia: The adult population density varies between 0 in winter to 2 in summer.
- In Senegal: the population increases from 2050 in winter to 2200 in summer.

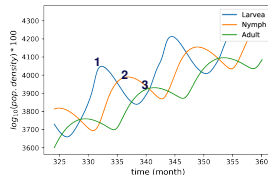
# Inactivity period is longer as we move northward



CA-1901: coexistence of the three different stages in summer and a long period of inactivity due to the long tick hibernation.



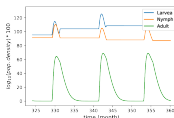
TN-1901: delay between larvae and adults and shorter periods of inactivity



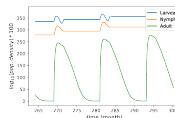
SN-1901: *Amblyomma variegatum* is active all along the year, the activity peaks of the three tick instars are chronologically well separated.

**Hibernation implies the coexistence of stages.**

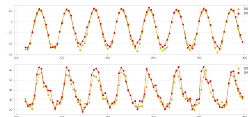
# Periode comparaison- CA



() CA-1901 – 1925



() CA-1991 – 2015

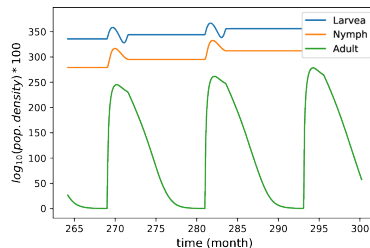
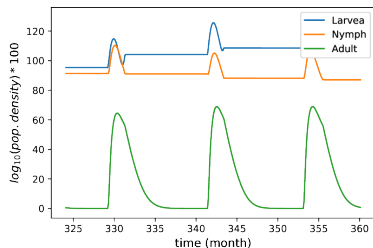


() Temp, pr

We showed an increase in the tick population in Canada due to an increase of precipitation.



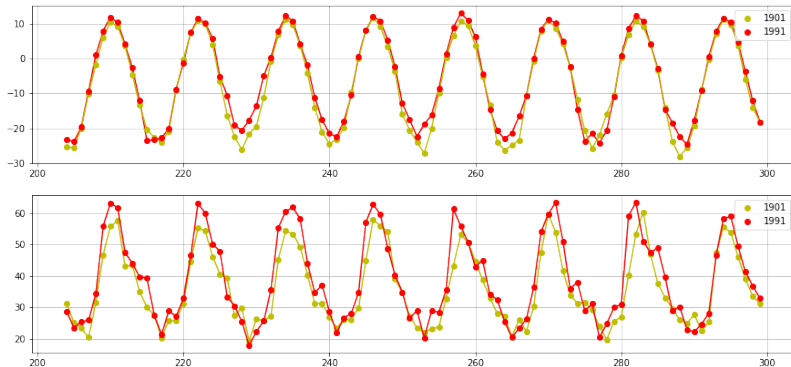
# Periode comparaison- CA



() CA-1901 – 1925    () CA-1991 – 2015

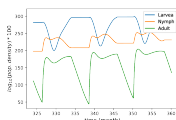
showed an increase in the tick population  
increase of precipitation.

# Periode comparaison- CA

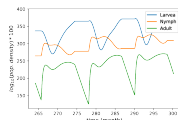


() Temp, pr

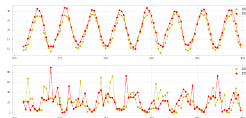
# Periode comparaison- TN



() TN-1901 – 1925



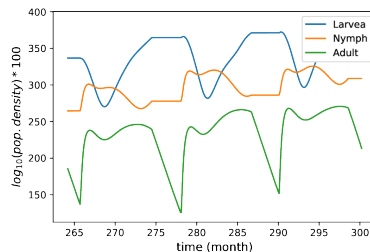
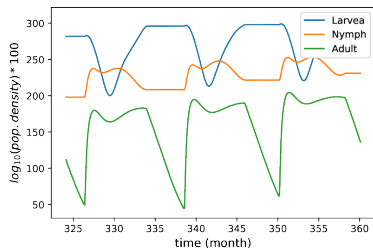
() TN-1991 – 2015



() Temp, pr

We showed an increase in the tick population in Tunisia due to both an increase of the max temperature and an increase of the periode of high temperature.

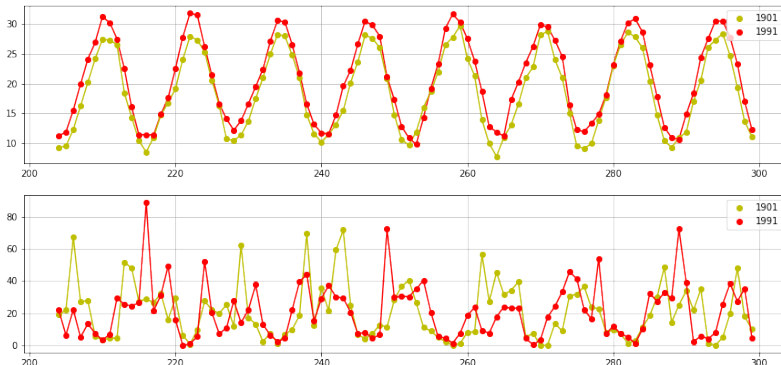
# Periode comparaison- TN



( ) TN-1901 – 1925    ( ) TN-1991 – 2015

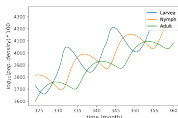
showed an increase in the tick population  
increase of the max temperature and a  
ht temperature.

# Periode comparaison- TN

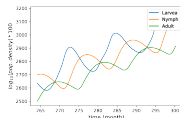


() Temp, pr

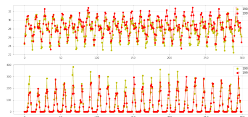
# Periode comparaison- SN



() SN-1901 – 1925



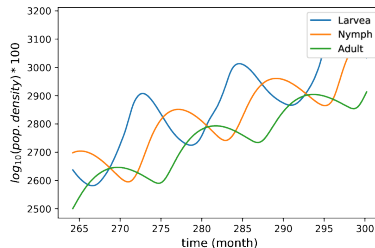
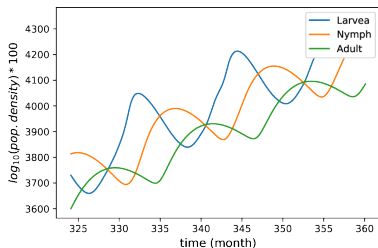
() SN-1991 – 2015



() Temp, pr

In the opposite, in Senegal the predicted decrease is related to the high precipitation in the beginning of 20th century.

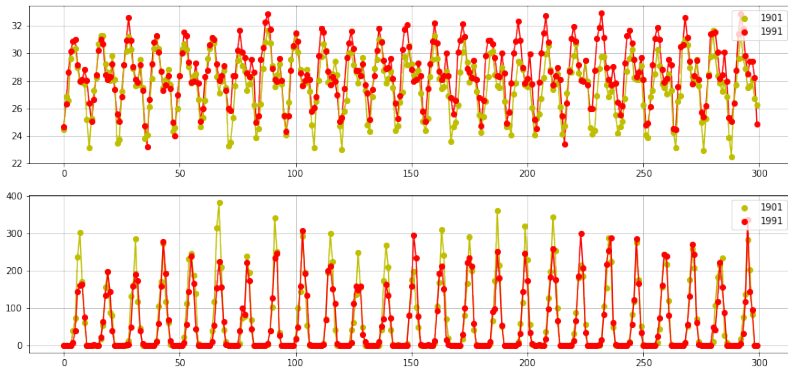
# Periode comparaison- SN



( ) SN-1901 – 1925    ( ) SN-1991 – 2015

he opposite, in Senegal the predicted c  
h precipitation in the begining of 20th c

# Periode comparaison- SN



() Temp, pr



According to our model, **larva**, **nymphs** and **adult** are predicted to be **present along the year** at constant densities outside the periods of density peaks, and **adult** are expected to be present over a **long period exceeding their usual activity season**.

These discrepancies are probably consecutive to the intended simplification of our model:

- Assumption that larva and nymphs mortalities are constant.
- The absence of consideration of others factors affecting free stages tick survival in the environment [Barre2010a].

We should remain out that our goal is to develop the less complex but enough informative model that could be applied to assess the global effects of temperatures and precipitations evolution under climate change on ticks population development.

Our model revealed globally for the period 1995 – 2015 comparatively to 1901 – 1925, a clear **trend for increased** tick densities in Canada (e.g. *I. cookie*) and in Tunisia (e.g. *H. scupense*). Opposite results were obtained in Senegal (e.g. *A. variegatum*).

Accordingly, temperature and precipitations overall changes from 1901 – 25 to 1995 – 2015 are potentially more favorable to tick development in Canada and Tunisia,

**Tick populations** may continue **to expand** if this dynamic of **climatic change is maintained** and if other tick population regulating factors are not coming into play.

Our model revealed globally for the period 1995 – 2015 comparatively to 1901 – 1925, a clear **trend for increased** tick densities in Canada (e.g. *I. cookie*) and in Tunisia (e.g. *H. scupense*). Opposite results were obtained in Senegal (e.g. *A. variegatum*).

Accordingly, temperature and precipitations overall changes from 1901 – 25 to 1995 – 2015 are potentially more favorable to tick development in Canada and Tunisia,

**Tick populations** may continue **to expand** if this dynamic of **climatic change is maintained** and if other tick population regulating factors are not coming into play.

In Senegal our simulations are showing that tick population is **expanding infinitely** within the two periods (1901 – 25 and 1995 – 2015).

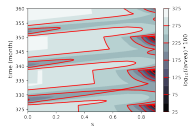
This feature is associated to the **absence of clear development** slowing under tropical climate.

Therefore, in Senegal the **regulation of tick populations** is probably depending on other regulating factors which are probably more important than in Canada and Tunisia, **role of biotic factors** *e.g.* tick predators [Stachurski2010] and **hosts genetic resistance** [Barre2010].

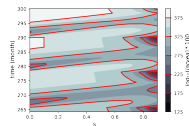
• Thank you for your attention

slimane.benmiled@fst.utm.tn

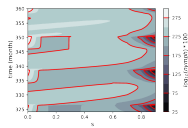
We then simulate the temporal variations in population densities in relation to the physiological structure and for each class, larvae, nymphs and adults.



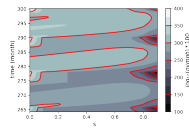
() CA-1901 – 1925



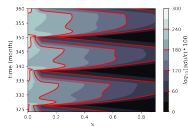
() CA-1991 – 2015



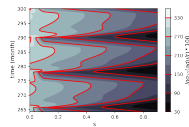
() TN-1901 – 1925



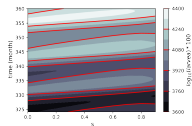
() TN-1991 – 2015



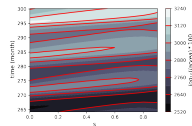
() SN-1901 – 1925



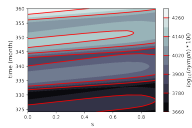
() SN-1991 – 2015



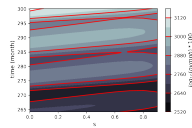
() CA-1901 – 1925



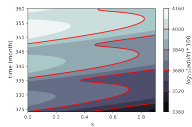
() CA-1991 – 2015



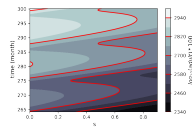
() TN-1901 – 1925



() TN-1991 – 2015



() SN-1901 – 1925



() SN-1991 – 2015