A DYNAMICAL MODEL FOR SYMPATRIC SPECIATION IN AN ECOLOGICAL NICHE

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Ecological niche

- An ecological niche β is composed by different interacting populations-species which live together and fit under specific environmental conditions.
- In general a niche β has a *core* or *heart* $\alpha \subset \beta$ and a *periphery* $\beta - \alpha$.
- See i.e. : Poleschova J, Storch D., (2019); Townsend Peterson et al. (2011) ; Chase J.M., Leibold M.A. (2003)

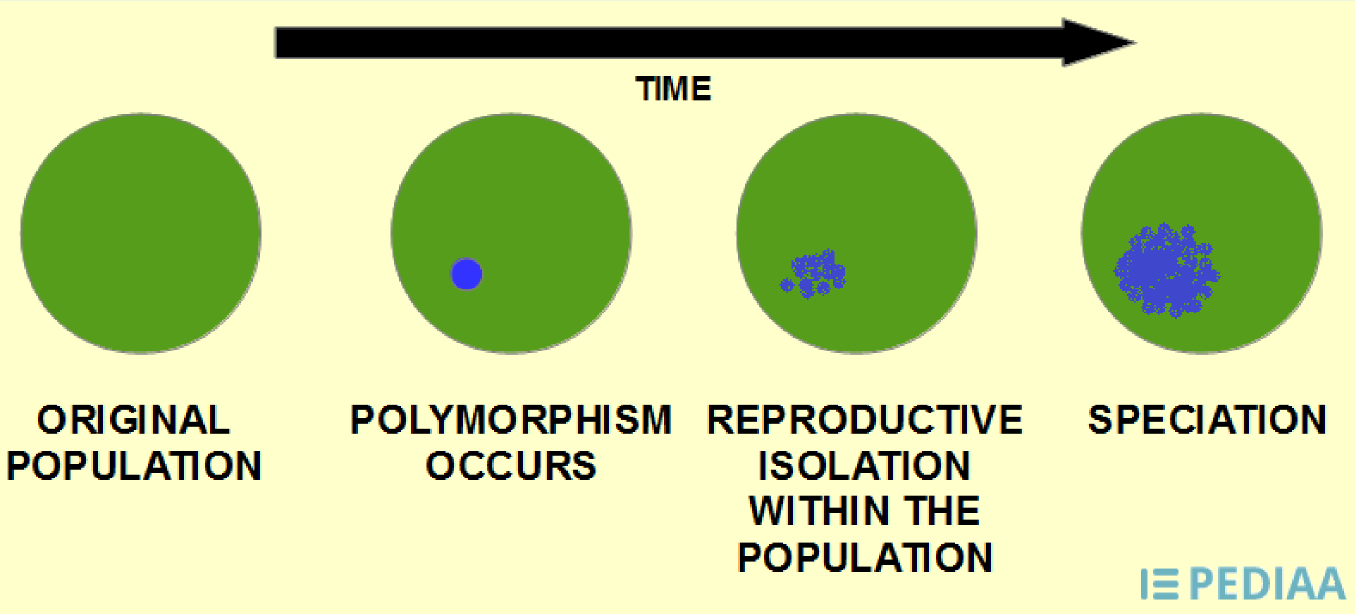
Ecological transformations and Sympatric speciation Model

- According to Maynard Smith the ecological transformations (*gene flow*) into the niche from the core (as sub-niche) to the periphery (as sub-niche) establishes a *polymorphism* for the considered population.

$$x_1(\text{in } \alpha) \Rightarrow x_2(\text{in } \beta\text{-}\alpha) \Rightarrow y(\text{in } \beta\text{-}\alpha)$$

- This natural differentiation between the two phenotypes x_1 and x_2 can lead to a sympatric speciation (that is, to have a new species y)

Sympatric speciation

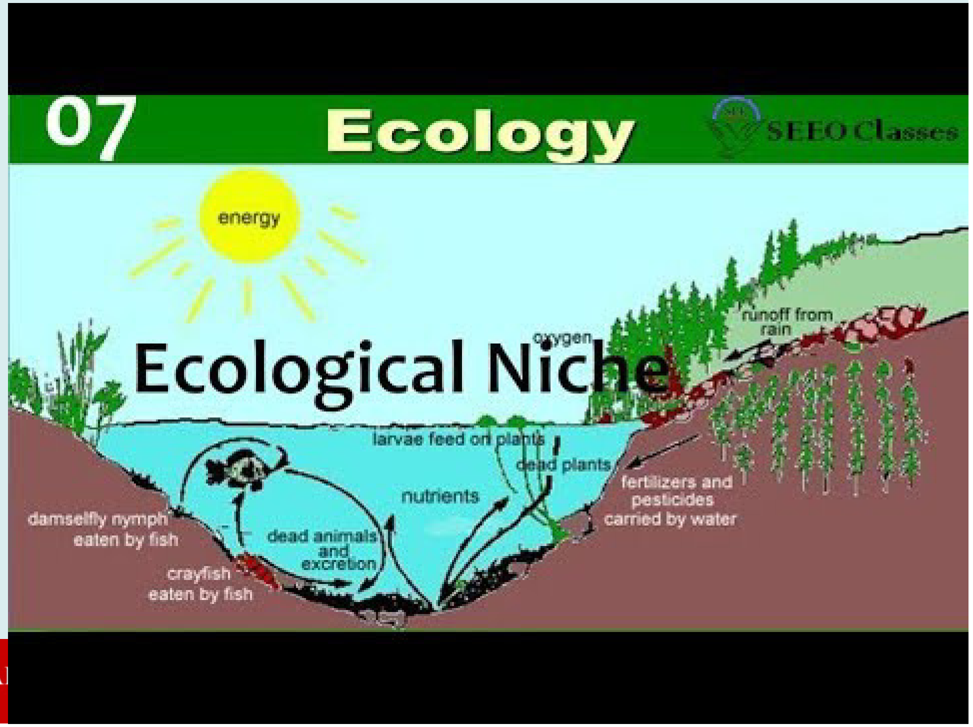


J.Maynard Smith (1966) formulated a model based on the existence of a diploid population with different viability selection in two niches.

Examples of niches and sympatric speciation

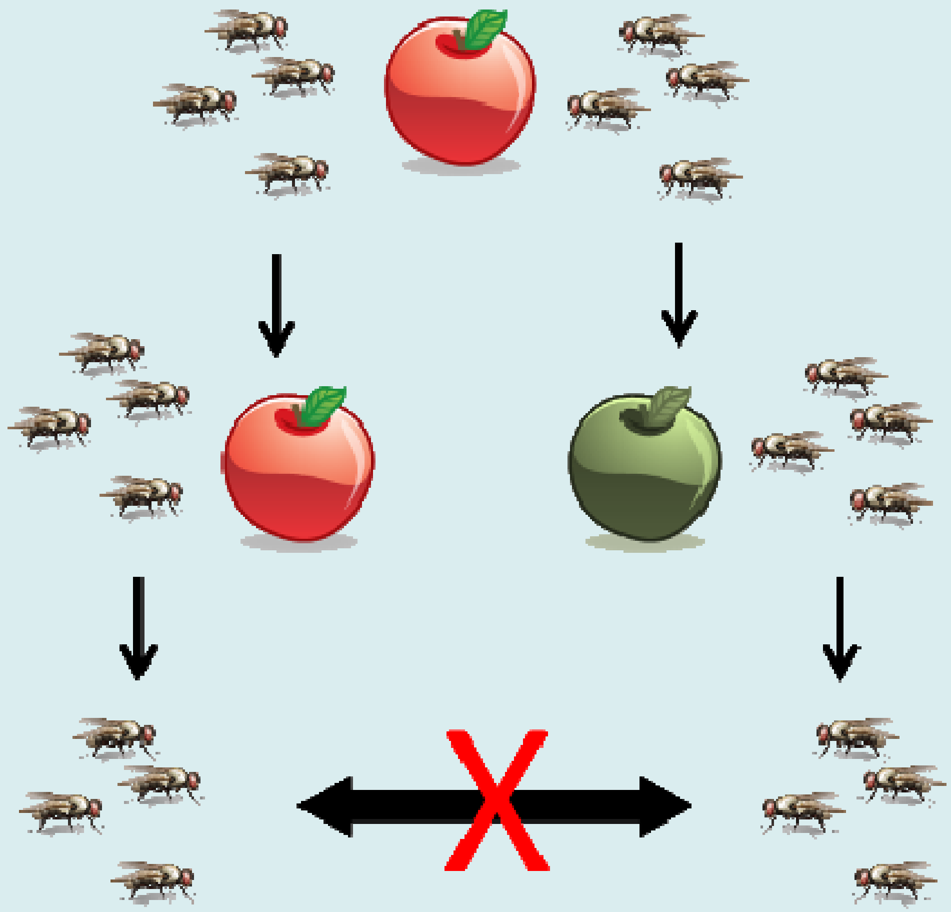
R.Lewontin (1983) describes the different heights (phenotypes) of the same plants of *Achillea*, grown at different altitudes . The two levels of altitude define core (α) and the periphery (β – α)

A lot of plants which live near (β – α) to a lake (α) i.e. *Cattail*, *Pickerehweed* etc. can grow with different heights.



Example of sympatric speciation in a particular niche

The hawthorn fly (*Rhagoletis pomonella*) is an example of sympatric speciation. The first niche isthe hawthorn tree (α) the extended niche is the apple tree (β - α). See Ernst Mayr and Guy Bush (1969)



Main assumptions at the base of population dynamic model

- The ancestral population has the possibility to express two phenotypes;
- In an ecological niche (finite resource environment) the populations tend to a stationary state due to mating behaviour;
- One phenotype has the possibility to isolate in an assortative mating process;
- The environment influences the population by random effects.

Preliminaries for modeling

- According to the population dynamics approach, based on the logistic equation [see i.e. C.H.Waddington (1959) and R.Lewontin (1983)], introduce a population x which assumes two different phenotypes x_1, x_2 such that: $x_1+x_2 = x \leq X^*$ (X^* is the finite *capacity* of the niche).
- The mutual interaction between the populations is defined by two parameters (μ and v).
- A parameter g gives the birth rate of the population and we introduce a threshold θ under which the population is not able to reproduce.

Two populations model (1)

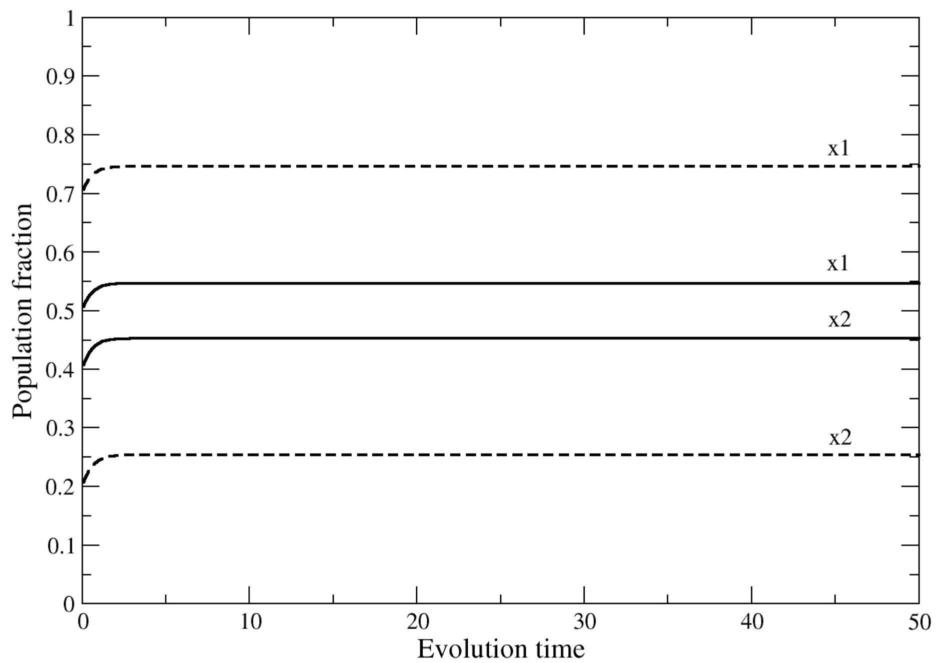
$$\frac{dx_1}{dt} = gx_1\left(1 - \frac{x_1}{X^* - x_2}\right) - \mu(X^* - x_1 - x_2)x_1 + \nu(X^* - x_1 - x_2)x_2$$

$$\frac{dx_2}{dt} = gx_2\left(1 - \frac{x_2}{X^* - x_1}\right) + \mu(X^* - x_1 - x_2)x_1 - \nu(X^* - x_1 - x_2)x_2$$

The two populations are coupled and they reach stationary state with the constraint:

$x_1 + x_2 \leq X^*$.

The stationary solution depends from the initial conditions.

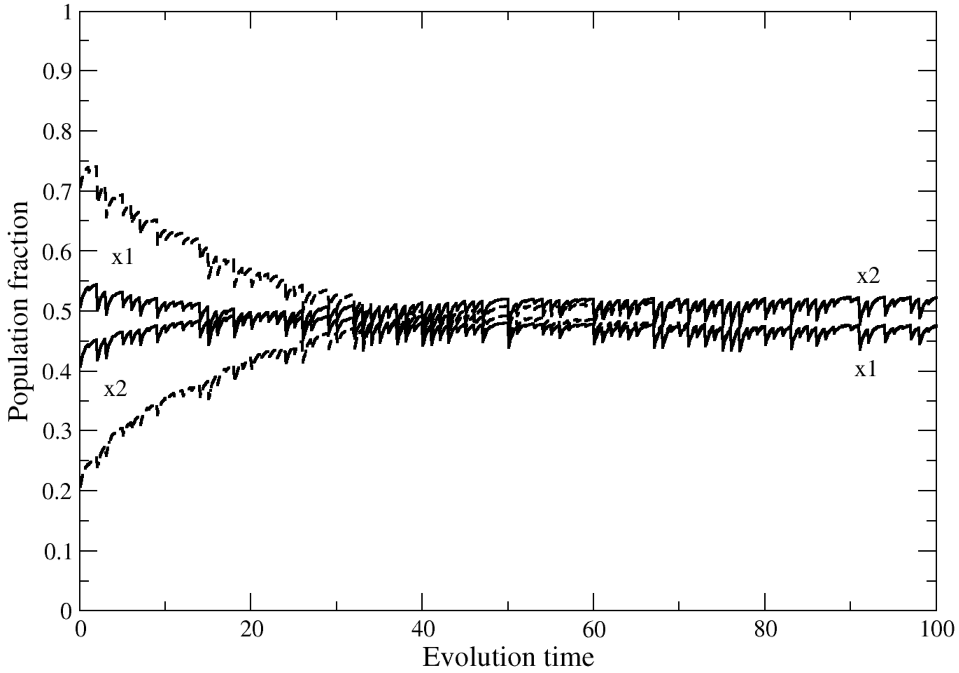


Effects of enviromental changes Model (2)

$$\frac{dx_1}{dt} = gx_1\left(1-\frac{x_1}{X^*-x_2}\right) - \mu(X^*-x_1-x_2)x_1 + \nu(X^*-x_1-x_2)x_2 - \epsilon x_1 \sum_{k\geq 0}\delta(t-kT)\eta_k$$

$$\frac{dx_2}{dt} = gx_2\left(1-\frac{x_2}{X^*-x_1}\right) + \mu(X^*-x_1-x_2)x_1 - \nu(X^*-x_1-x_2)x_2 - \epsilon x_1 \sum_{k\geq 0}\delta(t-kT)\eta_k$$

We introduce random fluctuations with Dirac δ in the populations with a time scale T and amplitude $\varepsilon = .1$. The stationary populations are uniquely defined. $T = g$ in the simulations and η_k are i.i.d. random variables, $\eta_k \in [0,1]$.



Existence of assortative mating Model (3)

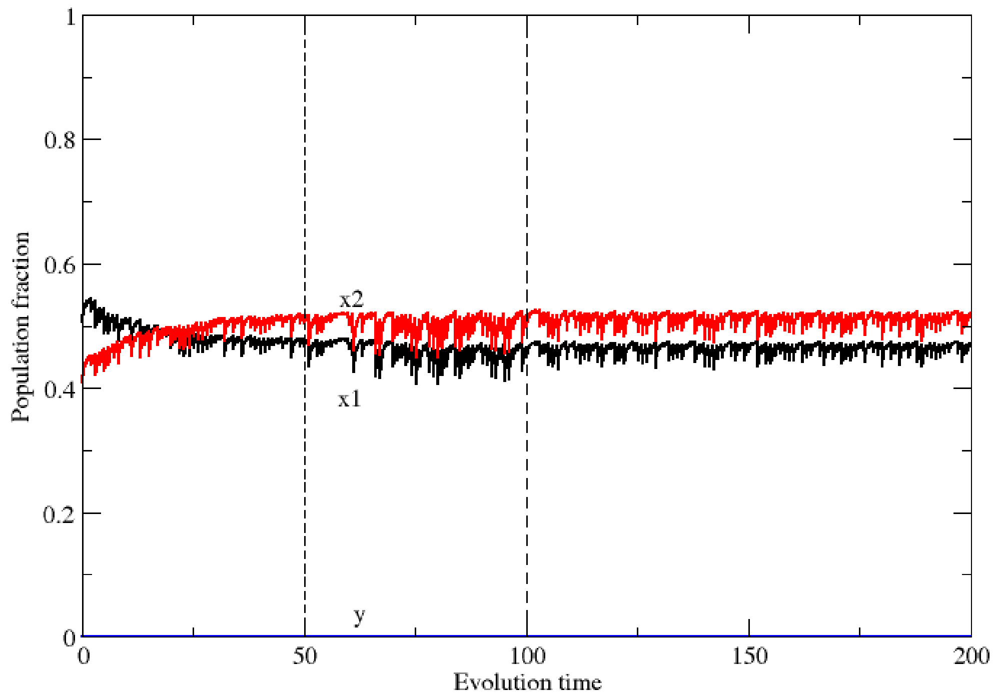
$$\frac{dx_1}{dt} = (g-\theta^*)x_1\left(1-\frac{x_1}{X^*-(x_2+y)}\right) - \mu(X^*-x_1-x_2-y)x_1 + \nu(X^*-x_1-x_2-y)x_2 - \epsilon x_1\sum_{k\geq 0}\delta(t-kT)\eta_k$$

$$\frac{dx_2}{dt} = (g-\theta^*)x_2\left(1-\frac{x_2}{X^*-(x_1+y)}\right) + \mu(X^*-x_1-x_2-y)x_1 - \nu(X^*-x_1-x_2-y)x_2 - \sigma(X^*-x_1-x_2-y)x_2 - \epsilon x_2\sum_{k\geq 0}\delta(t-kT)\eta_k$$

$$\frac{dy}{dt} = (g-\theta^*)\left(1-\frac{y}{X^*-(x_1+x_2)}\right) + \sigma(X^*-x_1-x_2-y)x_2 - \epsilon y\sum_{k\geq 0}\delta(t-kT)\eta_k$$

The new population y could grows up from the phenotype x_2 according to a mechanism of assortative mating: i.e. the new population y tends to isolate in the reproduction process starting the speciation process.

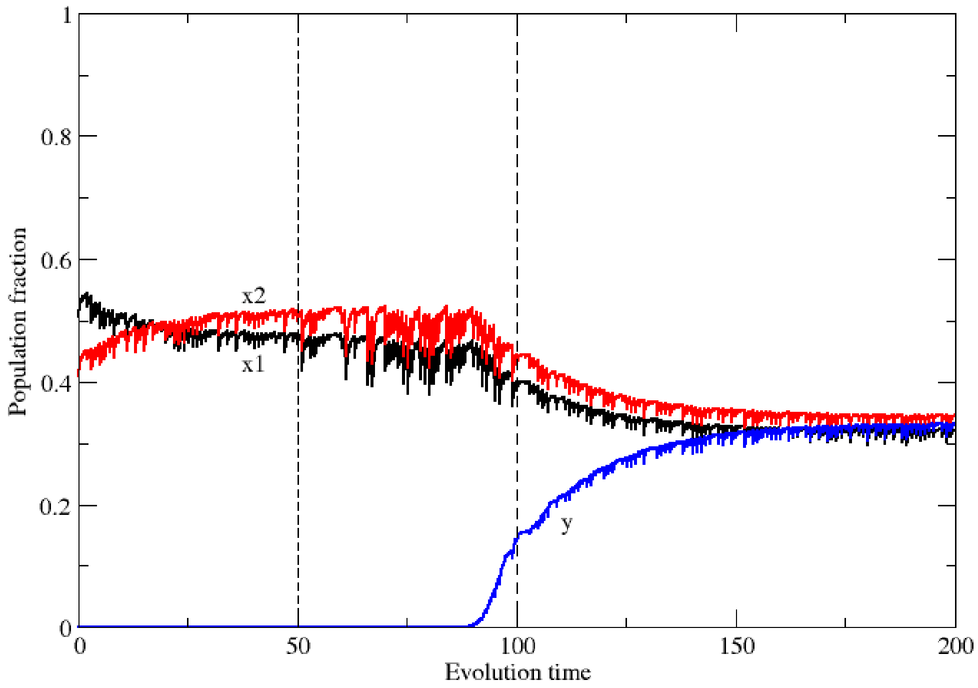
Effect of environmental fluctuations



Non stationary enviromental fluctuations: we assume an increase of the fluctuation amplitude $\varepsilon = .15$ for a limited period.

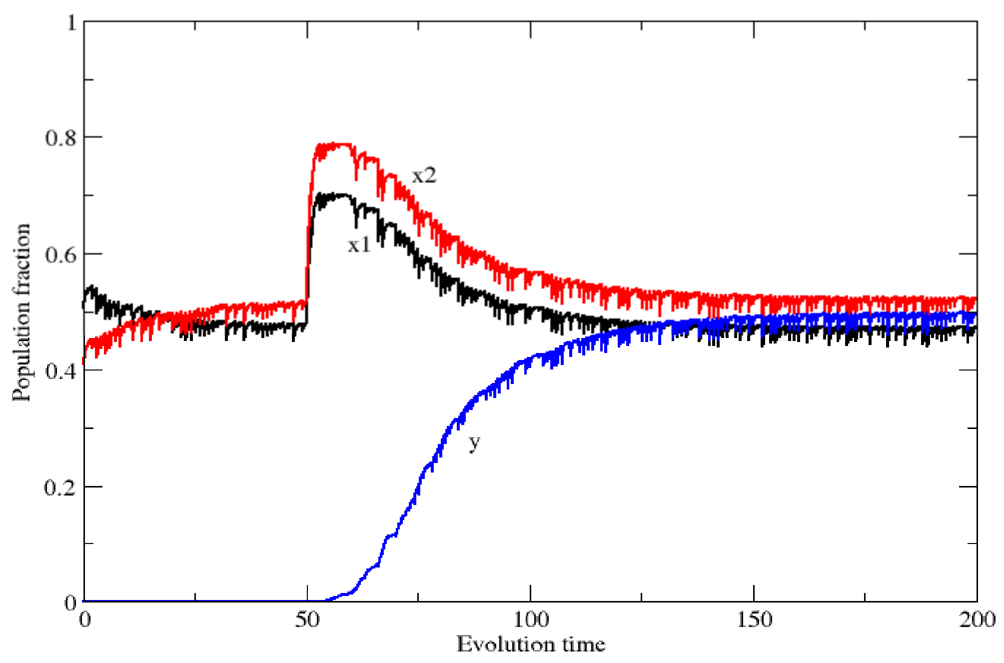
No speciation is observed.

Effect of environmental fluctuations



We increase the environmental stress by an amplitude $\varepsilon = .2$ for a period. The new population y appears in the niche and a new stationary state is reached (the populations have the same fitness).

Effect of a change in the niche



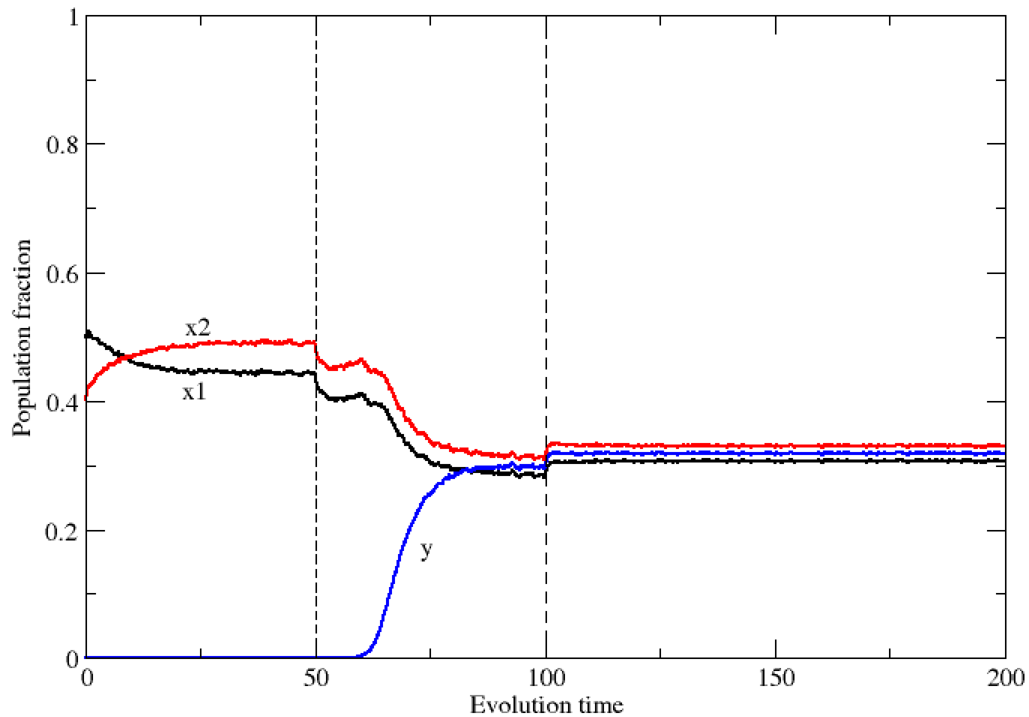
We keep fixed the fluctuations but the niche capacity is suddenly increased and the new population *y* appears.

The Model (4)

A further model is given by means of Ito stochastic differential equations, whose noise is additive, hence no more driven by a Dirac delta, as in (3). This kind of noisy term makes the effect of stochasticity more globally distributed over the time window. The diffusive part of the equation is governed by a single Wiener process.

$$dx_1(t) = (\text{same drift as in Model 3})\, dt - \varepsilon x_1^2 dW(t),$$
$$dx_2(t) = (\text{same drift as in Model 3})\, dt - \varepsilon x_2 y dW(t),$$
$$dx_3(t) = (\text{same drift as in Model 3})\, dt - \varepsilon x_2 y dW(t).$$

Numerical simulations of Model (4) with Wiener noise



Simulation of the solution to Model (4), using stochastic differential equations

Some conclusions

- For the sympatric speciation, our main assumptions are the existence of a population that can express two different phenotypes and the presence of stochastic fluctuations in the populations due to the changes in the environment.
- The model is formulated as a population dynamical model which considers the interaction between the initial populations and the probability of speciation in a new population.
- The existence of a threshold can be mathematically explained by an exponential dependence of the probability of realizing the speciation from the model parameters.
- Finally, the model shows as a selection process that favors the population with a greatest fitness, would allow to the new species to invade the niche even if the initial populations are still surviving.