Delay Induced Multiple Stability Switch and Chaos in a Predator-prey Model with Fear Effect

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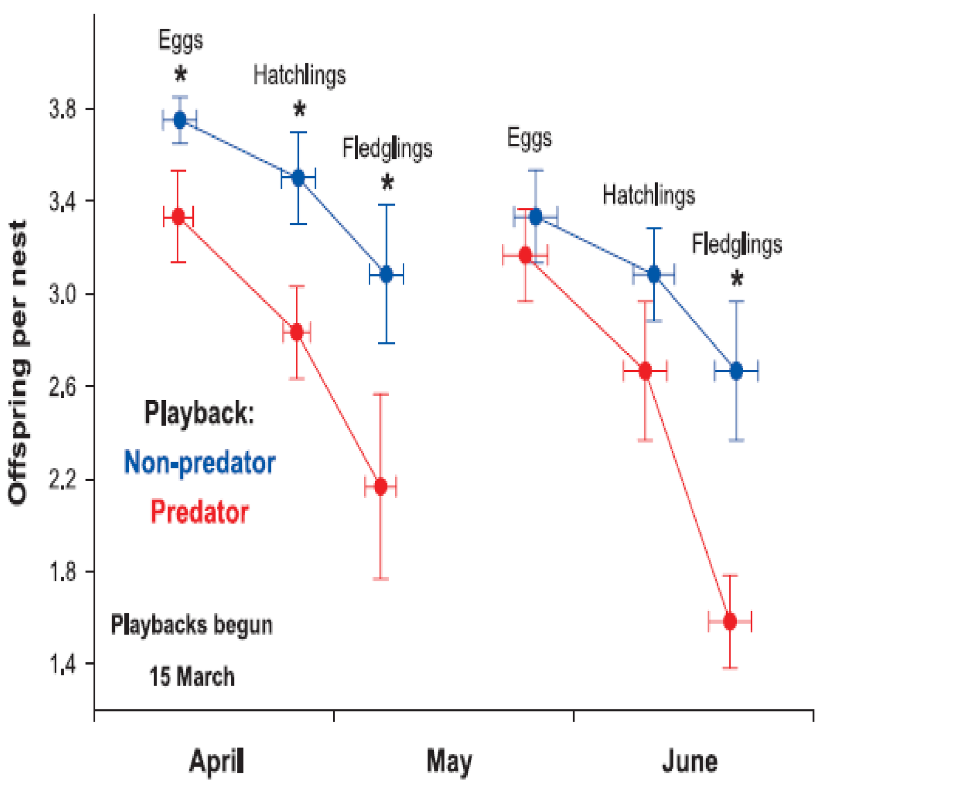
MOTIVATIONS

Predators impact on prey population in two ways:

- **Traditional approach**: Direct killing of preys.
- **Recent emerging view**: Induce fear on prey animals.

Due to predation fear, prey shows some anti-predator behaviour which may have cost on preys individual fitness.

Zanette et al. have experimentally showed only fear of predator through predator call and sound in a breeding season significantly reduces the number of offspring by 40%.



Zanette et al. 2011 (Science)

After sensing chemical or vocal cue of predator prey takes some time for assessing predation risk. So there must be some time lag between perceiving predation risk and reduction of growth rate of prey species.

MODEL ASSUMPTIONS

- Predator predates prey population and induce predation fear on prey population.
- Due to predation risk prey animal shows some anti-predator responses which reduces their growth rate.
- The growth reduction function is a monotonic decreasing function of predator density and level of predator fear.
- There must be some time lag between perceiving predation risk and reduction of growth of prey individual.

MATHEMATICAL MODEL

We formulate a predator-prey model incorporating fear induced delay by set of delay differential equations

$$\frac{dx}{dt} = \frac{x(1-x)}{1+ky(t-\tau)} - \frac{axy}{1+bx},$$

$$\frac{dy}{dt} = \frac{axy}{1+bx} - dy,$$

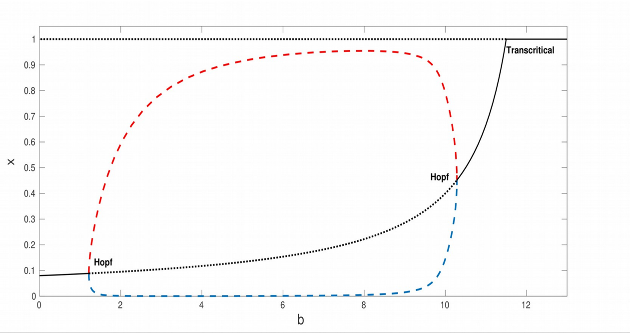
- x: prey population density at time t.
- y: predator population density at time t.
- a: maximum predation rate of predator.
- b: half saturation constant of predator.
- d: natural mortality rate of predator .
- k: level of fear of predator's which derives anti-predator behaviour of prey.
- τ time delay parameter.

IMPORTANCE OF WORK

- To understand the role fear of predator on ecosystem stability .
- To investigate the role of fear induce time delay on the dynamics of the system.

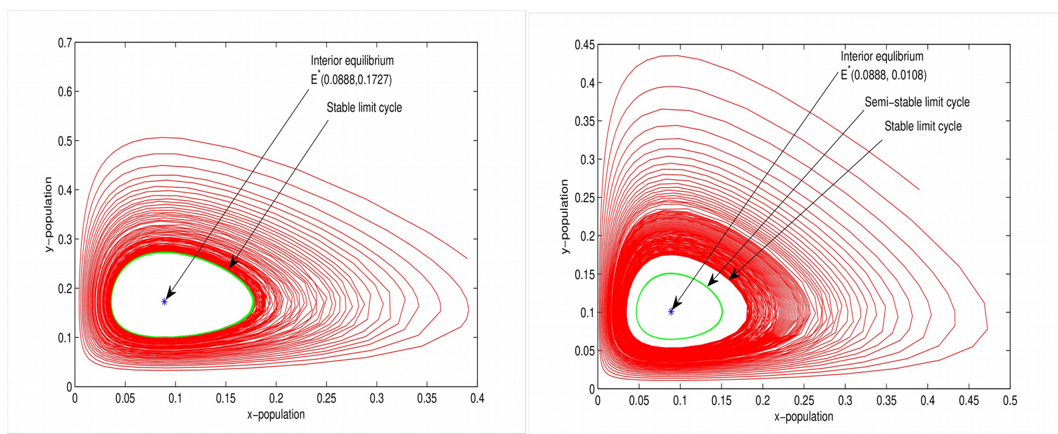
NUMERICAL SIMULATIONS (Non-delay model)

Bubbling Effect



Bifurcation diagram of non-delay model with bifurcation parameter b and other parameters are a=5, d=0.4, k=1.

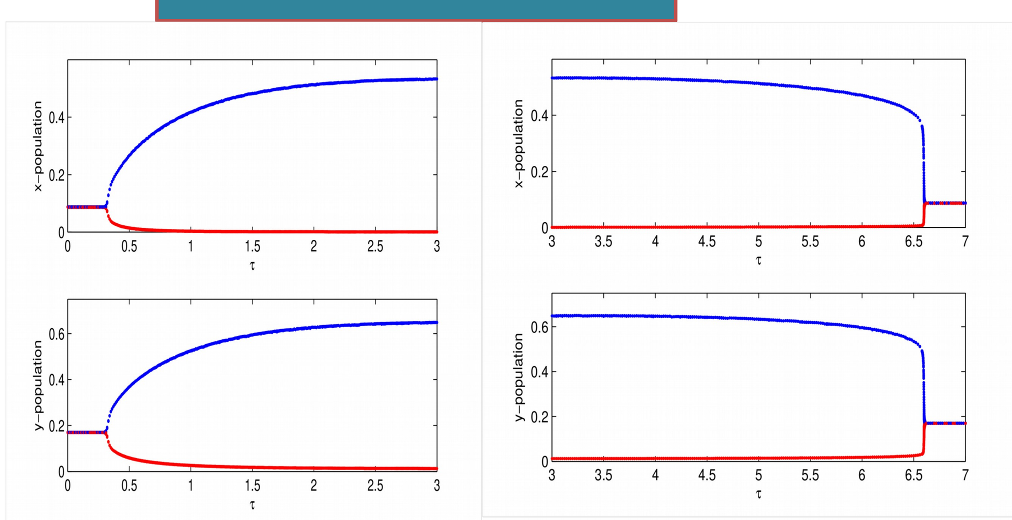
Existence of Stable and Semi-stable limit cycle



Corresponding to different initial conditions left figure represents stable limit cycle behaviour of the system for k=1, and right figure shows stable and semi-stable limit cycle of the system for k=10. Here other parameters are a=5, d=0.4, b=1.25.

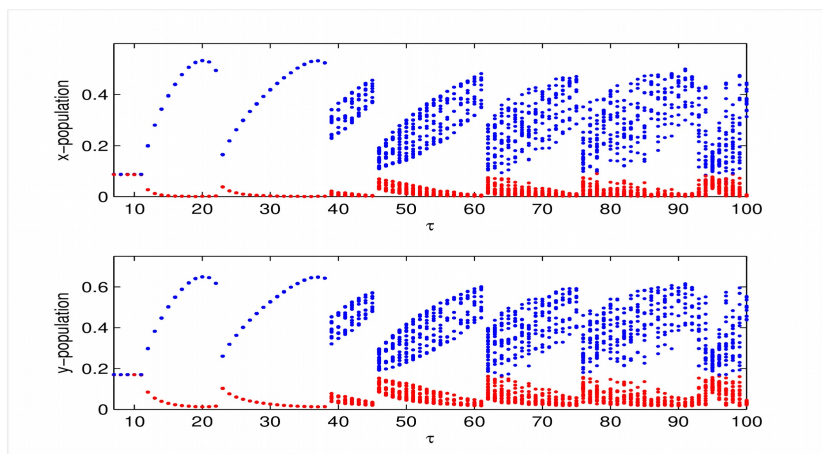
NUMERICAL SIMULATIONS (Delay model)

Switching Dynamics



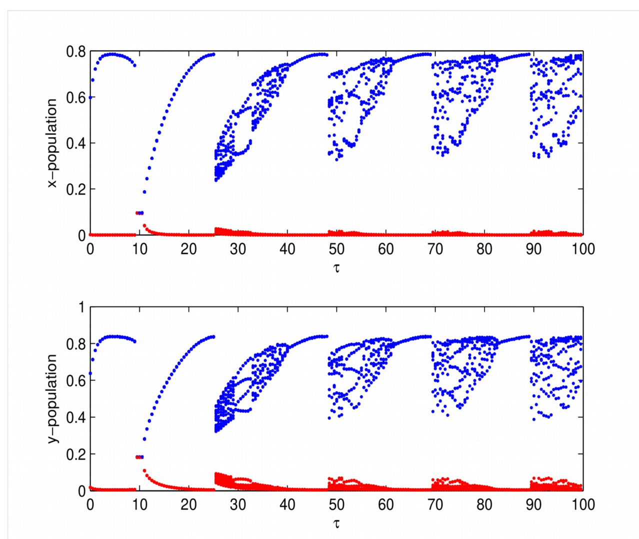
Bifurcation diagram of the system w.r.t. τ , where b=1 and other parameters are same as above figure. Switching dynamics is occur with increasing value of τ.

Multiples Switching and Chaos

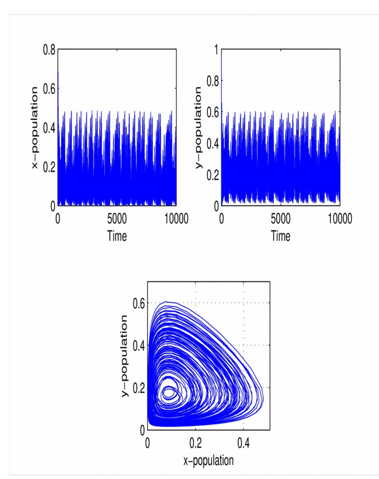


(a)

Fig. (a) Bifurcation diagram of the system w.r.t. τ where τ belongs to [7, 100] and a=5, b=1, d=0.4, k=1. After multiple switching of dynamics the system enters into chaotic regime.



(b)

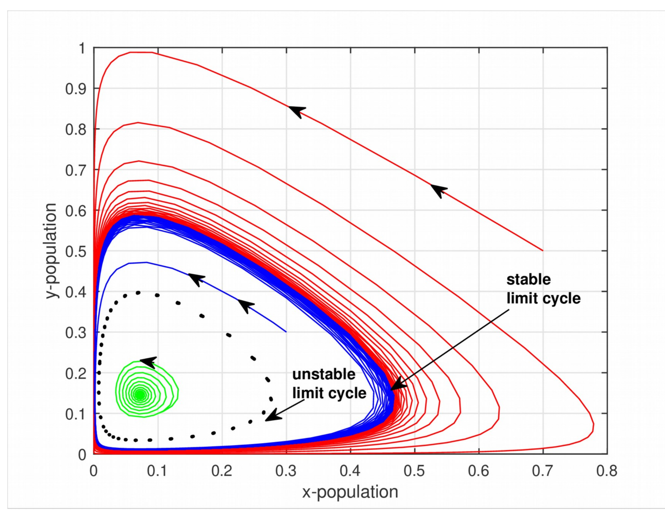


(c)

Fig. (b) Bifurcation diagram of the system w.r.t. τ where τ belongs to [0, 100] and a=5, b=1.5, d=0.4, k=1 . Fig. (c) Time series and phase diagram of the system for τ =100.

NUMERICAL SIMULATIONS (contd…)

Bi-stability



Bi-stability behaviour of system where parameters value are a=6, b=1.1, d=0.4, k=1, τ =7. Red and blue trajectories start form two different initial points (0.7, 0.5) and (0.3, 0.3) and converges to a limit cycle, where green trajectories start from (0.1, 0.1) and converges to a equilibrium point.

CONCLUSIONS

- For non delay model fear factors can change the stability properties of limit cycle.
- Fear induce delay has both stabilizing and destabilizing effects depending on the magnitude of delay parameter.
- Large magnitude of delay can produces chaotic dynamics of the system .
- Delay system also exhibits node-cycle bi-stability behaviour.

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