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IDEAL FLUIDS OF OPTIMAL FORAGES

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Diel vertical migration is considered as one of the largest daily movements of marine species where animals remain in deep, dark water during daylight hours to avoid visual predation and migrate to upper levels at dusk to feed. The migration of each organism can be rationalized as a trade-off between growth and survival with strategies as spatial distributions of the populations. The dynamics driving vertical migration has broad implications for fluxes through the food-web predator-pray interactions and how biomass is transported from lower to higher trophic levels [1, 4]; for vertical transport of carbon from upper to deeper layers (i.e. the so-called "biological carbon pump") with implications for global climate study [2].

Here, we present the idea for expressing diel vertical migration as a "vertical game" in terms of partial differential equations and show preliminary results of analysis of these equations. In the model setup we consider a population of animals distributed over the water column, and assume that each player (an animal) in this game moves optimally, seeking regions which offer high growth rates and small mortality, penalizing formation of regions with high density of the population. Recent results either doesn't take into account cost of movement [4] or doesn't resolve time continuously [1]. Here, we formulate the problem in continuous time and incorporate costs on excessive movements in our model. We show that the Nash equilibrium for this mean field game [3] is characterized by partial differential equations, which govern the distributions and migration velocities of animals. The derived system of PDEs has similarities to equations that appear in the fluid dynamics, specifically the Euler equations for compressible inviscid fluids. We show that if the environment is constant, the ideal free distribution emerges as an equilibrium. This equilibrium is hyperbolic, so that solutions can both converge to and depart from the ideal free distribution. We also present a discussion on computational approaches for solving this game theoretical model. We solve derived partial differential equations using spectral methods and present initial numerical results.

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