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FUNCTIONAL FOKKER-PLANCK EQUATIONS IN POPULATION BIOLOGY

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We review some predator-prey systems with complex bifurcation structures, where stochastic versions can be given easily. Namely, the Rosenzweig-MacArthur model with seasonal forcing shows Hopf and torus bifurcations, and then also bifurcations into chaotic attractors with positive Lyapunov exponents. Stochastic versions can be obtained by disentagling the time scale separation leading to the classical Rosenzweig-MacArthur model, where searching and handling predators as well as resource limitations for the prey via birth-death processes are considered. Markov processes and Fokker-Planck equations leading to stochastic differential equations are given and compared in their numerical performance.

Then we show some recently investigated mutator-replicator dynamics, with global or local trait specific resource limitation, in a similar way as done in the predator-prey systems mentioned above. Dynamics for local expectation values and Fokker-Planck equations can be derived in a similar way as above, and in limiting cases of continuous trait space, functional Fokker-Planck equations show up. These mutator-replicator systems are closely related to well known processes in evolutionary contexts.

Finally we will give some implications for the investigation of dynamic multi-strain epidemiological systems, i.e. where no explicit static fitness functions have to be given, with discrete or continuous trait space. Further implications are given for data analysis of empirical systems, since we have readly the stochastic formulations obtained.

References

[1] Stollenwerk, N., & Jansen, V. (2011) *Population Biology and Criticality: From critical birth–death processes to self-organized criticality in mutation pathogen systems* (Imperial College Press, World Scientific, London)

[2] Stollenwerk, N, Fuentes Sommer, P., Kooi, B., Mateus, L., Ghaffari, P., & Aguiar, M. (2017) Hopf and torus bifurcations, torus destruction and chaos in population biology, *Ecological Complexity*, **30**, 91–99.

[3] Kimura, M. (1965) A stochastic model concerning the maintanance of genetic variability in quantitative characters, *Proc. Natl. Acad. Sci. USA* **54**, 731–736.

[4] Kolmogorov, A., Petrovski, I., & Piscounov, N. (1937) A study of the diffusion equation with increase in the amount of substance, and its application to a biological problem, *Moscow Univ. Bull. Ser. Internat. Sec.* **A 1**, 1–25.

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[5] Fisher, R.A. (1937) The wave of advance of advantageous genes, *Annals of Eugenics* **7**, 353–369.

[6] Aguiar, M., Ballesteros, S., Kooi, B.W., & Stollenwerk, N. (2011) The role of seasonality and import in a minimalistic multi-strain dengue model capturing differences between primary and secondary infections: complex dynamics and its implications for data analysis, *Journal of Theoretical Biology*, **289**, 181–196.

[7] Stollenwerk, N., & Jansen, V. (2003, a) Meningitis, pathogenicity near criticality: the epidemiology of meningococcal disease as a model for accidental pathogens, *Journal of Theoretical Biology*, **222**, 347–359.

[8] Stollenwerk, N., & Jansen, V. (2003, b) Evolution towards criticality in an epidemiological model for meningococcal disease. *Physics Letters A* **317**, 87–96.

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